# NATIONAL INSTITUTE OF TECHNOLOGY TIRUCHIRAPPALLI - 620015

**DEPARTMENT OF MATHEMATICS**

**SYLLABUS FOR WRITTEN TEST**

Calculus: Functions of two or more variables, continuity, directional derivatives, partial derivatives, total derivative, maxima and minima, saddle point, method of Lagrange's multipliers; Double and Triple integrals and their applications to area, volume and surface area; Vector Calculus: gradient, divergence and curl, Line integrals and Surface integrals, Green's theorem, Stokes' theorem, and Gauss divergence theorem.

Linear Algebra: Finite dimensional vector spaces over real or complex fields; Linear transformations and their matrix representations, rank and nullity; systems of linear equations, characteristic polynomial, eigenvalues and eigenvectors, diagonalization, minimal polynomial, Cayley-Hamilton Theorem, Finite dimensional inner product spaces, Gram-Schmidt orthonormalization process, symmetric, skew-symmetric, Hermitian, skew-Hermitian, normal, orthogonal and unitary matrices; diagonalization by a unitary matrix, Jordan canonical form; bilinear and quadratic forms.

Real Analysis: Metric spaces, connectedness, compactness, completeness; Sequences and series of functions, uniform convergence, Ascoli-Arzela theorem; Weierstrass approximation theorem; contraction mapping principle, Power series; Differentiation of functions of several variables, Inverse and Implicit function theorems; Lebesgue measure on the real line, measurable functions; Lebesgue integral, Fatou's lemma, monotone convergence theorem, dominated convergence theorem.

Complex Analysis: Functions of a complex variable: continuity, differentiability, analytic functions, harmonic functions; Complex integration: Cauchy's integral theorem and formula; Liouville's theorem, maximum modulus principle, Morera's theorem; zeros and singularities; Power series, radius of convergence, Taylor's series and Laurent's series; Residue theorem and applications for evaluating real integrals; Rouche's theorem, Argument principle, Schwarz lemma; Conformal mappings, Mobius transformations.

Ordinary Differential Equations: First order ordinary differential equations, existence and uniqueness theorems for initial value problems, linear ordinary differential equations of higher order with constant coefficients; Second order linear ordinary differential equations with variable coefficients; Cauchy-Euler equation, method of Laplace transforms for solving ordinary differential equations, series solutions (power series, Frobenius method); Legendre and Bessel functions and their orthogonal properties; Systems of linear first order ordinary differential equations, Sturm's oscillation and separation theorems, Sturm-Liouville eigenvalue problems, Planar autonomous systems of ordinary differential equations: Stability of stationary points for linear systems with constant coefficients, Linearized stability, Lyapunov functions.

Algebra: Permutations, combinations, pigeon-hole principle, inclusion-exclusion principle, derangements. Fundamental theorem of arithmetic, divisibility in Z, congruences, Chinese Remainder Theorem, Euler's Ø- function, primitive roots.

Groups, subgroups, normal subgroups, quotient groups, homomorphisms, automorphisms; cyclic groups, permutation groups, Group action, Sylow's theorems and their applications; Rings, ideals, prime and maximal ideals, quotient rings, unique factorization domains, Principle ideal domains, Euclidean domains, polynomial rings, Eisenstein's irreducibility criterion; Fields, finite fields, field extensions, algebraic extensions, algebraically closed fields.

Functional Analysis: Normed linear spaces, Banach spaces, Hahn-Banach theorem, open mapping and closed graph theorems, principle of uniform boundedness; Inner-product spaces, Hilbert spaces, orthonormal bases, projection theorem, Riesz representation theorem, spectral theorem for compact self-adjoint operators.

Numerical Analysis: Systems of linear equations: Direct methods (Gaussian elimination, LU decomposition, Cholesky factorization), Iterative methods (Gauss-Seidel and Jacobi) and their convergence for diagonally dominant coefficient matrices; Numerical solutions of nonlinear equations: bisection method, secant method, Newton-Raphson method, fixed point iteration; Interpolation: Lagrange and Newton forms of interpolating polynomial, Error in polynomial interpolation of a function; Numerical differentiation and error, Numerical integration: Trapezoidal and Simpson rules, Newton-Cotes integration formulas, composite rules, mathematical errors involved in numerical integration formulae; Numerical solution of initial value problems for ordinary differential equations: Methods of Euler, Runge-Kutta method of order 2.

Partial Differential Equations: Method of characteristics for first order linear and quasilinear partial differential equations; Second order partial differential equations in two independent variables: classification and canonical forms, method of separation of variables for Laplace equation in Cartesian and polar coordinates, heat and wave equations in one space variable; Wave equation: Cauchy problem and d'Alembert formula, domains of dependence and influence, non- homogeneous wave equation; Heat equation: Cauchy problem; Laplace and Fourier transform methods.

Topology: Basic concepts of topology, bases, subbases, subspace topology, order topology, product topology, quotient topology, metric topology, connectedness, compactness, countability and separation axioms, Urysohn's Lemma.

Linear Programming: Linear programming models, convex sets, extreme points; Basic feasible solution, graphical method, simplex method, two phase methods, revised simplex method ; Infeasible and unbounded linear programming models, alternate optima; Duality theory, weak duality and strong duality; Balanced and unbalanced transportation problems, Initial basic feasible solution of balanced transportation problems (least cost method, north-west corner rule, Vogel's approximation method); Optimal solution, modified distribution method; Solving assignment problems, Hungarian method.

Probability: Axiomatic definition of probability, properties of probability function, conditional probability, Bayes' theorem, independence of events; Random variables and their distributions, distribution function, probability mass function, probability density function and their properties, expectation, moments and moment generating function, quantiles, distribution of functions of a random variable, Chebyshev, Markov and Jensen inequalities.

Standard discrete and continuous univariate distributions: Bernoulli, binomial, geometric, negative binomial, hypergeometric, discrete uniform, Poisson, continuous uniform, exponential, gamma, beta, Weibull and normal.

Jointly distributed random variables and their distribution functions, probability mass function, probability density function and their properties, marginal and conditional distributions, conditional expectation and moments, product moments, simple correlation coefficient, joint moment generating function, independence of random variables, functions of random vector and their distributions, multinomial distribution, bivariate normal distribution and sampling distributions.

Stochastic Processes: Markov chains with finite and countable state space, classification of states, limiting behaviour of n-step transition probabilities, stationary distribution, Poisson process, birth-and-death process, pure-birth process, pure-death process, Brownian motion and its basic properties.