# MASTER OF SCIENCE IN MATHEMATICS (M.Sc., Mathematics)

Syllabus

for

Credit Based Flexible Curriculum

(From the academic year 2019-20 onwards)

# **BOARD OF STUDIES**



Department of Mathematics

National Institute of Technology Tiruchirappalli,

Tiruchirappalli – 620015

Tamil Nadu, India

# Vision, Mission, Core Values and Goals

NIT Tiruchirappalli, through its Vision, Mission and Core Values, defines herself as:

- An Indian institution with world standards
- A global pool of talented students, committed faculty and conscientious researchers
- Responsive to real-world problems and, through a synergy of education and research, engineer a better society

## VISION

• To be a university globally trusted for technical excellence where learning and research integrate to sustain society and industry.

## **MISSION**

- To offer undergraduate, postgraduate, doctoral and modular programmes in multi-disciplinary / inter-disciplinary and emerging areas.
- To create a converging learning environment to serve a dynamically evolving society.
- To promote innovation for sustainable solutions by forging global collaborations with academia and industry in cutting-edge research.
- To be an intellectual ecosystem where human capabilities can develop holistically.

## **Core Values:**

## Integrity

Honest in intention, fair in evaluation, transparent in deeds and ethical in our personal and professional conduct that stands personal and public scrutiny.

## Excellence

Commitment to continuous improvement coupled with a passion for innovation that drives the pursuit of the best practices; while achievement is always acknowledged, merit will always be recognized.

#### Unity

Building capacity through trust in others' abilities and cultivating respect as the cornerstone of collective effort.

#### Inclusivity

No one left behind; no one neglected; none forgotten in the mission of nation-building through higher learning.

## Goals:

International Accreditation and Ranking in tertiary education largely guide goal-setting. The perception built by the stakeholders, crucially influence the process of repositioning. Benchmarking with global universities who are in the top 200 in world rankings in terms of teaching, innovation and research, funding and internationalization. Hence, the need to set the following goals:

- Attracting top talent and global collaborations
- Building world-class research infrastructure to facilitate multi- / inter- / trans-disciplinary research
- Initiatives towards financial sustainability
- Social outreach activities of national / international importance
- Top 10 in India ranking in Engineering Discipline

Top 500 in World Ranking in five years

#### THE DEPARTMENT OF MATHEMATICS

The Department of Mathematics is one of the pioneering and the most distinguished departments in National Institute of Technology, Trichy. Applying a multi-disciplinary research and teaching methods, the department strongly believes in finding mathematical solutions for various social-economic, technological and work related processes and challenges. With fourteen faculty members representing major areas of mathematics, the department is at the forefront of cutting-edge research as well as teaching and innovation.

The department is committed to outstanding graduate training to produce leading scholars in various fields of mathematics. Since its inception, the department molds Ph. D graduates to carry out challenging research problems which have wide ranging industrial and social implications. Students are provided with ample opportunities to improve their research, teach courses, and participate in conferences/seminars.

#### Vision

 To be a global Centre of Excellence in mathematics and scientific computing for the growth of science and technology.

#### Mission

 Committed to the cause of quality education, research and consultancy by providing principled and highly skilled mathematics.

#### MASTER OF SCIENCE IN MATHEMATICS

## Program Educational Objectives (PEOs)

- 1. Graduates will contributes rapidly growing multidisciplinary research that uses advanced computing capabilities to understand and solve complex problems.
- 2. Graduate of the programme will be capable of handling every problem existing around the world through mathematical structures.
- 3. Graduate of the programme will become competent users of mathematics and to provide mathematical solution to real life problems.
- 4. Graduates will continue lifelong learning and pursue higher studies in mathematical and statistical sciences

## Program Outcome: Graduate will be able to

- a. Progress the critical analysis and problem solving skills required for research and development organization and industry.
- b. Communicate confidently and effectively with industry and society at large, regarding complex problem and solution of the problem, existing around.
- c. Engage independent and lifelong learning with a high level of enthusiasm and commitment to improve knowledge and competence continuously.
- d. Contribute significantly in academics through teaching and research.
- **e.** Demonstrate knowledge and understanding of various structure of mathematics and apply the same to one's own work, as a member and leader in a team, manage projects efficiently after consideration of economical and financial factors.
- **f.** Apply ethical principles and commit to professional ethics and responsibilities and norms of the professional practice.

The board of studies for Master of Science in Mathematics of department of mathematics includes the following members:

#### Chairman

• Dr. P. Saikrishnan Head, Department of Mathematics

## **External Experts**

- Dr. Satyajit Roy, Professor
  Department of Mathematics,
  Indian Institute of Technology, Madras
  Chennai, Tamil Nadu
- Dr. A. Raja, General Manager, AMAR Tech, Bengalure-560004 (Siemons Centre of Excellence, NIT Trichy)
- Dr. A. Raghunathan, Additional General Manager, BHEL, Trichy

#### Members

- Dr. R. Ponalagusamy
- Dr.D. Deivamoney Selvam
- Dr. K. Murugesan
- Dr. T.N.Janakiraman
- Dr. V. Kumaran
- Dr. V. Ravichandran
- Dr. R. Tamil Selvi
- Dr. V. Lakshmana Gomathi Nayagam
- Dr. V. Shanthi
- Dr.I. Jeyaraman
- Dr. N. Prakash
- Dr. Jitraj Saha
- Dr. Vamsinadh Thota

## Course curriculum

# Semester-1

| S.No | Code    | Course of study                        | L | T | P     | Credit |
|------|---------|--|---|---|-------|--------|
| 1    | MA 7 01 | Real Analysis                          | 3 | 0 | 0     | 3      |
| 2    | MA 7 03 | Linear Algebra                         | 3 | 0 | 0     | 3      |
| 3.   | MA 7 05 | <b>Probability and Statistics</b>      | 3 | 0 | 0     | 3      |
| 4.   | MA 7 07 | <b>Ordinary Differential Equations</b> | 3 | 0 | 0     | 3      |
| 5.   | MA 7 09 | Programming language - C               | 3 | 0 | 0     | 3      |
| 6    | MA 7 11 | Programming in C-lab                   | 0 | 0 | 4     | 2      |
|      |         |  |   |   | Total | 17     |

## Semester-2

| S.No | Code    | Course of study                | L | T | P     | Credit |
|------|---------|--------------------------------|---|---|-------|--------|
| 1    | MA 7 02 | Algebra                        | 3 | 0 | 0     | 3      |
| 2    | MA 7 04 | Complex Analysis               | 3 | 0 | 0     | 3      |
| 3.   | MA 7 06 | Topology                       | 3 | 0 | 0     | 3      |
| 4.   | MA 7 08 | Partial Differential Equations | 3 | 0 | 0     | 3      |
| 5.   | MA 7 10 | Numerical Analysis             | 3 | 0 | 0     | 3      |
| 6    | MA 7 12 | Statistical Computing-Lab      | 0 | 0 | 4     | 2      |
|      |         |                                |   |   | Total | 17     |

## Semester-3

| S.No | Code     | Course of study            | L | T | P     | Credit |
|------|----------|----------------------------|---|---|-------|--------|
| 1    | MA 7 13  | <b>Functional Analysis</b> | 3 | 0 | 0     | 3      |
| 2    | MA 7 15  | Transforms Techniques      | 3 | 0 | 0     | 3      |
| 3.   | MA 7 17  | <b>Operations Research</b> | 3 | 0 | 0     | 3      |
| 4.   | ** ** ** | Elective - 1               | 3 | 0 | 0     | 3      |
| 5.   | ** ** ** | Elective – 2               | 3 | 0 | 0     | 3      |
| 6    | MA 7 19  | Mathematical software Lab  | 0 | 0 | 4     | 2      |
|      |          |                            |   |   | Total | 17     |

# Semester-4

| S.No | Code     | Course of study          | L | T | P     | Credit |
|------|----------|--------------------------|---|---|-------|--------|
| 1    | ** ** ** | Elective 3               | 3 | 0 | 0     | 3      |
| 2    | ** ** ** | Open Elective/Elective 4 | 3 | 0 | 0     | 3      |
| 3    | MA 7 14  | Project                  | 0 | 0 | 0     | 8      |
|      |          |                          |   |   | Total | 14     |

Total credits: 65

# List of electives

| S.No | Code    | Course of study                             | L | T | P | Credit |
|------|---------|---|---|---|---|--------|
| 1    | MA 7 24 | Integral equations & Calculus of Variations | 3 | 0 | 0 | 3      |
| 2    | MA 7 26 | Non-Linear programming                      | 3 | 0 | 0 | 3      |
| 3    | MA 7 16 | Hardy-Hilbert Spaces                        | 3 | 0 | 0 | 3      |
| 4    | MA 7 18 | Operator theory                             | 3 | 0 | 0 | 3      |
| 5    | MA 7 21 | Graph Theory                                | 3 | 0 | 0 | 3      |
| 6    | MA 7 27 | Numerical solution of DE                    | 3 | 0 | 0 | 3      |
| 7    | MA 7 29 | Fluid dynamics                              | 3 | 0 | 0 | 3      |
| 8    | MA 7 23 | Measure Theory                              | 3 | 0 | 0 | 3      |
| 9    | MA 7 25 | Sobolov Spaces and its Applications         | 3 | 0 | 0 | 3      |
| 10   | MA 7 20 | Fuzzy Mathematics and its Applications      | 3 | 0 | 0 | 3      |
| 11   | MA 7 22 | Asymptotics and Perturbation Methods        | 3 | 0 | 0 | 3      |

| Course Code         | MA 7 01       |
|---------------------|---------------|
| Title of the Course | Real Analysis |
| Prerequisite        | NIL           |
| Credits (L-T-P)     | 3 (3-0-0)     |

#### Course Learning Objectives: This course

- 1. introduces various concepts related to real numbers, differentiation and integration.
- 2. also deals with mean value theorems and convergence of series of functions. In addition, an introduction to metric spaces is also given.

#### **Course Content**

Infimum, supremum and limit point of a subset of real numbers. liminf, limsup and limit of a sequence of real numbers. Nature of series of real numbers. Limit, continuity, differentiation and Riemann integration of real valued functions. Riemann-Stieltjes Integral, existence of the integral. Condition for integrability, properties, integral as a limit of a sum, first mean value theorem, Second mean value theorem. The Riesz representation theorem.

Sequences and series of real valued functions, pointwise convergence, uniform convergence, Cauchy's criterion and test for uniform convergence of sequence of functions. Tests for uniform convergence of series of functions (Weierstrass's M-test, Abel's test, Dirichlet's test). Uniform convergence versus continuity (Dini's theorem), integration and differentiation. The Weierstrass approximation theorem.

Metric spaces, basic concepts, Cauchy's sequence and convergence of a sequence in metric spaces. Complete metric spaces. Connectedness, intermediate value theorem. Separable metric spaces. Compactness, Heine-Borel theorem. Continuous and uniformly continuous functions from one metric space to other. The Banach Contraction Principle. Continuous functions on a metric space. Homeomorphisms, Equivalent metrics. Completion of a metric space. Equicontinuous family of functions. The Arzela-Ascoli theorem, The Baire Category Theorem.

#### **Reference Books:**

- 1. N. L. Carothers, Real Analysis, Cambridge University Press, 2000
- 2. H. L. Royden, P. M. Fitzpatrick, Real Analysis, 4th ed., Pearson education, 2011
- 3. W. Rudin, Principles of Mathematical Analysis, Mc-Graw Hill, 1976
- 4. G. F. Simmons, Introduction to and Modern Analysis, Kreiger Publishing Co., 1983

- 1. find liminf, limsup and discuss continuity and differentiability of functions
- 2. define Riemann- Stieltjes integral, evaluate them for various functions and understand their properties.
- 3. test convergence of series of functions.
- 4. understand the various concepts of topology in the metric space setting.

| Course Code         | MA 7 03        |
|---------------------|----------------|
| Title of the Course | Linear Algebra |
| Prerequisite        | NIL            |
| Credits (L-T-P)     | 3 (3-0-0)      |

- discuss various decompositions of vector spaces and linear transformations on vector spaces.
- study diagonalizable operator on a vector space and characterizations of it using the minimal and characteristic polynomials.
- introduce different classes of linear operators on inner product spaces and to study their structures.
- learn the concepts of bilinear and quadratic forms on vector spaces.

#### **Course Content**

Review of basic concepts: Vector spaces, Bases, Dimension, Linear Transformations - Characteristic values and characteristic vectors - Diagonalization - Eigenspaces - Minimal polynomial - Cayley-Hamilton Theorem.

Invariant subspaces – Direct-sum Decompositions – Invariant Direct sums – The Primary Decomposition Theorem.

Cyclic subspaces – Cyclic Decomposition Theorem – Rational and Jordan forms – Computation of invariant factors.

Basic review of Inner Product Spaces – Adjoint operators – Normal operators – Unitary Operators – Orthogonal projections – The spectral Theorem.

Bilinear forms – Matrix representation – Quadratic forms – Sylvester's law of inertia – Principal Axis Theorem - Positive definite forms.

#### **Reference Books:**

- 1) Kenneth Hoffman and Ray Kunze, "Linear Algebra", PHI, 2010.
- 2) Stephen H. Friedberg, Arnold J. Insel and Lawrence E. Spence, "*Linear Algebra*", PHI, 2013.
- 3) Sheldon Axler, "Linear Algebra Done Right", Springer, 1997.
- 4) Steven Roman, "Advanced inear Algebra", Springer, 2008.

- 1. find characteristic values, characteristic vectors and the minimal polynomial of a linear transformation and to determine a linear transformation is diagonalizable.
- 2. decompose a vector space into a sum of invariant subspaces and a linear transformation into a direct sum of induced operators.
- 3. compute the cyclic subspace generated by a vector and to construct the rational and Jordan forms of linear transformations and matrices.
- 4. determine a linear operator is normal, unitary and orthogonal projection and to construct the spectral decomposition of normal and self-adjoint operators.
- 5. construct the matrix of a bilinear form and to find index, rank and signature of a bilinear form.

| Course Code         | MA 7 05                    |
|---------------------|----------------------------|
| Title of the Course | PROBABILITY AND STATISTICS |
| Prerequisite        | Nil                        |
| Credits (L-T-P)     | 3 (3-0-0)                  |

- understand and use the probability concepts in formulating and study real life situation.
- provide a solution for real life problems using the various elements probability, probability density function, moments, probability calculation etc.
- apply the concepts of estimation theory, random sampling, test statistical hypotheses in solving many real life applications for further improvement and modification.

#### **Course Content**

Definitions of probability - Probability spaces- Random variables- Probability Mass and Density functions- Discrete and Continuous distributions - Standard and non-standard types.

Mathematical expectation- Generating functions- Probability and Moment generating functions- Characteristic function – Two variables - Joint distribution function - Conditional density and Expectations – Covariance - Coefficient of correlation - Multiple random variables and its moments.

Chebyshev's - Markov type inequalities- Convergences in probability— Law of large numbers- Central limit theorem- Applications.

Fundamental concepts in statistics- Measures of location and variability- Population, sample, parameters- Point and interval estimation- Method of moments, Maximum likelihood estimator, Properties of estimator, Unbiasedness, Consistency, Efficiency- Confidence intervals for mean, Difference of means, Proportions.

Testing of hypothesis - Null and alternate hypothesis -Neyman Pearson fundamental lemma - Tests for one sample and two sample problems for normal populations - Tests for proportions - Test for small samples - t - test, chi - square test and F-test.

#### **Reference Books:**

- 1. William Feller: *An Introduction to Probability Theory and its Applications*, 3<sup>rd</sup> edition, Vol. I and Vol II, New York, Wiley India, 2008.
- 2. Kai Li Chung: A course in Probability Theory, 3<sup>rd</sup> edition, Academic Pres, 2001.
- 3. S. M. Ross: Introduction to Probability Models, 11th edition, Academic Press, 2014.
- 4. Robert V. Hogg, J.W. McKean, and Allen T. Craig: *Introduction to Mathematical Statistics*, 7<sup>th</sup> Edition, Pearson Education, Asia, 2014.
- 5. Edward J Dudewicz and Satya N. Mishra: *Modern Mathematical Statistics*, International Edition, Wiley. 1988.

- understand the axiomatic rudiments of modern Probability theory and use of random variables as an intrinsic tool for the analysis of random phenomena.
- characterize multiple input and output system probability models and function of random variables based on single & multiple random variables.
- evaluate and apply moments & characteristic functions and understand the concept of inequalities and probabilistic limits.
- able to use basic statistical knowledge in testing hypotheses on large and small samples and estimations.

| Course Code         | MA 7 07                         |
|---------------------|---------------------------------|
| Title of the Course | ORDINARY DIFFERENTIAL EQUATIONS |
| Prerequisite        | Nil                             |
| Credits (L-T-P)     | 3 (3-0-0)                       |

## Course Learning Objectives: This course introduces

- 1. various approach to find general solution of the ordinary differential equations
- 2. theorems to discuss the existence and uniqueness of solution of IVP for ODE
- 3. special functions and its properties

#### **Course Content**

The general solution of the homogeneous equation –The method of variation of parameters – Power Series solutions- Higher order linear equation-operator methods for finding particular solutions.

Series solutions of first order equations – Second order linear equations; Ordinary points. Regular Singular Points – Gauss's hypergeometric equation – The Point at infinity - Legendre Polynomials – Bessel functions – Properties of Legendre Polynomials and Bessel functions.

Linear Systems of First Order Equations – Homogeneous Equations with Constant Coefficients – The Existence and Uniqueness of Solutions of Initial Value Problem for First Order Ordinary Differential Equations – The Method of Solutions of Successive Approximations and Picard's Theorem.

Oscillation Theory and Boundary value problems – Qualitative Properties of Solutions – Sturm Comparison Theorems – Eigenvalues, Eigen functions and the Vibrating String.

Nonlinear equations: Autonomous Systems; the phase plane and its phenomena – Types of critical points; Stability – critical points and stability for linear systems – Stability by Liapunov's direct method – Simple critical points of nonlinear systems.

#### **Reference Books:**

- 1. E.A. Coddington, An Introduction to Ordinary Differential Equations, Courier Corporation, 2012
- 2. G.F. Simmons, Differential Equations with Applications and Historical Notes, CRC Press, 2016
- 3. M.E. Taylor, Introduction to Differential Equations, AMS Indian Edition, 2011.
- 4. William E. Boyce, Richard C. DiPrima, Douglas B. Meade, Elementary Differential Equations and Boundary Value Problems, Wiley, 2017.
- 5. Lawrence Perko, Differential Equations and Dynamical Systems, Springer Science & Business Media, 2013

- 1. find the solutions of first and some higher order ordinary differential equations
- 2. discuss the existence and uniqueness of solutions of first and second order ODE
- 3. apply properties of special functions in discussion the solution of ODE.
- 4. model some physical problem and give physical interpretation of the solution.

| Course Code         | MA 7 09          |
|---------------------|------------------|
| Title of the Course | Programming in C |
| Prerequisite        | Nil              |
| Credits (L-T-P)     | 3 (3-0-0)        |

## Course Learning Objectives: This course make the student to

- 1. understand and write programs in language of C for a given problems.
- 2. analyze the concepts of arrays and tables for storage
- 3. involve in creating files for the problems.
- 4. interpret the programs through pointers.

#### **Course Content**

**Introduction to C:** The C character set - Identifiers, Constants and keywords -Primitive datatypes - Operators and Expressions-Library functions- Data Input and Output.

**Control Statements**: Nested control structures - Functions-Function prototypes -Passing arguments to a function.

**Program Structure**: Storage classes –Arrays-Declaration, initialization, and accessing array elements- Arrays and strings.

**Pointers:** Pointer declarations -Passing pointers to a function -Pointers and one dimensional arrays -Dynamic memory allocation -Operations on pointers -Pointers and multidimensional arrays -Arrays of pointers -Passing functions to other functions.

**Structures and Unions**: Defining a structure -Processing a structure -User-defined datatypes(typedef) -Structures and pointers -Passing structures to functions -Self-referential structures. Data Files-Operations-Formatted input and output- Character input and output.

#### **Reference Books:**

- 1. B. S. Gottfried & J. K. Chhabra, Programming with C, Second Edition, Tata McGraw-Hill, New Delhi, 2006.
- 2. B.W. Kernighan & D. M. Ritchie, The C Programming Language, Second Edition, Prentice Hall of India Pvt. Limited, New Delhi, 2006.
- 3. V. Rajaraman, Computer Programming in C, Prentice Hall of India Pvt. Ltd. New Delhi, 2004.
- 4. E. Balagurusamy, Programming in ANSI C by, 7<sup>th</sup> Edition, Tata McGraw-Hill Publishing Co. Ltd., New Delhi, 2017.

- 1. write structured pseudo codes for a given problem.
- 2. design programs in C for any given problem.
- 3. manage writing programs for complex problems.
- 4. attain the capability of developing files through pointers.

| Course Code         | MA 7 11                      |
|---------------------|------------------------------|
| Title of the Course | Programming in C- Laboratory |
| Prerequisite        | Nil                          |
| Credits (L-T-P)     | 2 (0-0-2)                    |

## Course Learning Objectives: This course to

- 1. make the student learn a programming language.
- 2. learn problem solving techniques.
- 3. teach the student to write programs in C and to solve the problems.

#### **Course Content**

Implementing the concepts of programming language C: Basics, operators, Loop operations, Arrays, Math Functions and I/O Functions, Functions and Recursion, Structures, File operations using command line arguments.

## Reference Books:

- 1. Programming with C by B. S. Gottfried & J. K. Chhabra, Second Edition, Tata McGraw-Hill, New Delhi, 2006.
- 2. The C Programming Language by B.W. Kernighan & D. M. Ritchie, Second Edition, Prentice Hall of India Pvt. Limited, New Delhi, 2006.
- 3. Computer Programming in C by V. Rajaraman, Prentice Hall of India Pvt. Ltd. New Delhi, 2004.
- 4. Programming in ANSI C by E. Balagurusamy, Second Edition, Tata McGraw-Hill Publishing Co. Ltd., New Delhi, 2017.

- 1. read, understand and trace the execution of programs written in C language.
- 2. write the C code for a given algorithm.
- 3. develop programs for complex problems applying the concepts of Arrays and pointers.

| Course Code         | MA 7 02   |
|---------------------|-----------|
| Title of the Course | ALGEBRA   |
| Prerequisite        | Nil       |
| Credits (L-T-P)     | 3 (3-0-0) |

- 1. introduce the concepts of conjugacy classes and Sylow's theorems
- 2. explain the Fundamental Theorem of Finite Abelian Groups
- 3. learn the various types of integral domains
- 4. expose the students to extensions field and its properties
- 5. learn the Galois Theory and solvability.

## Course Content

Review of basic Group Theory – Group actions – Conjugacy classes – The class equation – Sylow's Theorem - Direct Product –Fundamental Theorem of Finite Abelian Groups.

Review of basic Ring Theory – Ideals and Factor rings – Prime and Maximal ideals—Euclidean domains—principal ideal domains and unique factorization domains—Polynomial rings – Factorization of Polynomials.

Extension fields – Splitting fields – Algebraic and Transcendental extensions – Simple extensions – Separable extensions - Finite fields.

Galois Theory – Fundamental Theorem of Galois Theory – Solvability of Polynomials by Radicals – Solvable groups – Insolvability of a quantic.

#### **Reference Books:**

- 1. D. S. Dummit and R. M. Foote: *Abstract Algebra*, 3<sup>rd</sup>Edition, John-Wiley, 2011.
- 2. M. Artin: *Algebra*, 2<sup>nd</sup> edition, Pearson, 2011.
- 3. I.N. Herstein: *Topics in Algebra*, 2<sup>nd</sup> edition, John-Wiley, 2008.
- 4. J.A. Gallian: *Contemporary Abstract Algebra*, 4<sup>th</sup>edition, Narosa, 1999.
- 5. N. Jacobson: Basic Algebra I and II, 2nd Edition, Dover Publication Inc., 2009.

- 1. analyze the concepts of conjugacy classes and Sylow's theorem
- 2. understand the properties of various type of integral domains
- 3. gain the knowledge on the extension fields
- 4. understand the concepts of Galois Theory and solvability

| Course Code         | MA 7 04          |
|---------------------|------------------|
| Title of the Course | Complex Analysis |
| Prerequisite        | Real Analysis    |
| Credits (L-T-P)     | 3(3-0-0)         |

## **Course Learning Objectives:**

The course presents an introduction to analytic functions, conformal mappings, Mobius transformations and power series. Various Cauchy' theorems are discussed and used in evaluation of integral. It deals with locations of zeros of analytic functions and maximum principles.

#### **Course Content**

Lines and planes in complex plane, extended complex plane, spherical representation, power series, analytic functions as mappings, branch of logarithm, conformal mappings, Mobius transformations.

Power series representation of analytic functions, zeros of analytic functions, index of a closed curve, Cauchy's theorem and integral formula on open subsets of C.

Homotopy, homotopic version of Cauchy's theorem, simple connectedness, counting of zeros, open mapping theorem, Goursat's theorem, Classification of singularities, Laurent series.

Residue, Contour integration, argument principle, Rouche's theorem, Maximum principle, Schwarz' lemma.

#### **Reference Books:**

- 1. Conway John. Functions of One Complex Variables. 2nd ed, Narosa, New Delhi. 2002.
- 2. Ahlfors Lars. Complex Analysis. McGraw Hill Co., New York. 1988.
- 3. Hahn Liang-Shin and Epstein Bernard. Classical Complex Analysis. Jones and Bartlett India, New Delhi. 2011.
- 4. Rudin Walter. Real and Complex Analysis. McGraw-Hill. 1987.
- 5. Ullrich David. Complex Made Simple. American Math. Soc., Washington DC. 2008.

- 1. understand analytic functions as mappings and discuss properties of conformal mappings, and Mobius transformations
- 2. obtain series representation of analytic functions
- 3. evaluate various integrals by using Cauchy's residue theorem
- 4. classify singularities and derive Laurent series expansion

| Course Code         | MA 7 06   |
|---------------------|-----------|
| Title of the Course | Topology  |
| Prerequisite        | Nil       |
| Credits (L-T-P)     | 3 (3-0-0) |

## Course Learning Objectives: This course

1. introduces the notion of open and closed sets and deals with continuity, connectedness, compactness and various countability and separation axioms.

#### **Course Content**

Finite, countable, uncountable sets. Functions and relations. Partially ordered sets, well ordered sets. Axiom of choice, Well-ordering theorem, The maximum principle, Zorn's lemma.

Topological spaces, open sets, closed sets. closure and interior of a set. Limit points of a set. Basis for a topology, subbasis. subspace topology, order topology, product topology  $X \times Y$ . T1, T2 and Hausdorff spaces, Metric topology.

Continuous functions, Homeomorphisms, constructing continuous functions, The pasting lemma. Product topology, box topology, quotient topology.

Connected spaces, components and locally connectedness, path connectedness. Compact spaces, limit point compactness, sequentially compactness, local compactness, finite intersection property. Compactifications.

Countability axioms, separation axioms, regular and normal spaces. The Urysohn's lemma, The Urysohn'smetrization theorem, The Tietze extension theorem, The Tychonoff theorem.

#### **Reference Books:**

- 1. J. R. Munkres, Topology, 2nd ed., Pearson Education India, 2001.
- 2. G. F. Simmons, Introduction to Topology and Modern Analysis, Kreiger Publishing Co., 1983
- 3. C. Wayne Patty, Foundations of Topology, Jones and Batlett, Delhi 2010.

- 1. understand various notions of topological spaces and derived concepts.
- 2. prove results about homeomorphism, product topology, connectedness and compactness.
- 3. prove theorems about Hausdorff spaces, Regular and Normal spaces.
- 4. construct new continuous functions and prove compactness in arbitrary product spaces.

| Course Code         | MA 7 08                        |
|---------------------|--------------------------------|
| Title of the Course | PARTIAL DIFFERENTIAL EQUATIONS |
| Prerequisite        | ODE                            |
| Credits (L-T-P)     | 3 (3-0-0)                      |

## Course Learning Objectives: This course

- 1. discuss various approach to find the solution of partial differential equations.
- 2. construct mathematical model and solution of some physical problem

#### **Course Content**

**FIRST ORDER EQUATIONS** Integral surfaces passing through a given curve - Surfaces orthogonal to a given system of surfaces - Compatible system of equations - Charpit's method.

**SECOND ORDER EQUATIONS** Classification of second order Partial Differential Equations - Reduction to canonical form - Adjoint operators. **HYPERBOLIC EQUATIONS** One-dimensional wave equation - Initial value problem - D' Alembert's solution - Riemann - Volterra solution - Vibrating string - Variables Separable solution - Forced vibrations - Solutions of Non-homogeneous equation - Vibration of a circular membrane.

**PARABOLIC EQUATIONS** Diffusion equation - Method of Separation of variables: Solution of one and two dimensional Diffusion equations in Cartesian coordinates and Solution of Diffusion equation in cylindrical and spherical polar coordinates.

**ELLIPTIC EQUATIONS** Boundary value problems - Properties of harmonic functions - Green's Function for Laplace Equation - The Methods of Images - The Eigen function of Method.

#### **Reference Books:**

- 1. I.N. Snedden, Elements of Partial Differential Equations, McGraw Hill, 1985.
- 2. T.Amarnath, An Elementary Course in Partial Differential Equations ,Narosa Publishing Company, 1997.
- 3. Phoolan Prasad and RenukaRavindran, Partial Differential Equations, Wiley-Eastern Ltd, 1985
- 4. LokenathDebnath and DambaruBhatta, Integral Transforms and Their Applications, Chapman & Hall/CRC; 2 edition, 2006.
- 5. TynMyint-U: Partial differential equations for scientists and engineers, 3rd ed. North Holland, 1989.
- 6. I.C. Evans, Partial Differential Equations, Graduate Studies in Mathematics, Vol. 19 AMS, 1998.

- 1. form the partial differential equation for family of surfaces.
- 2. find solution of Laplace equation for various boundary conditions.
- 3. model vibration of an elastic string/membrane and find discuss solution of it.
- 4. model one dimensional heat equation and find analytic solution for some boundary condition.

| Course Code         | MA 7 10            |
|---------------------|--------------------|
| Title of the Course | NUMERICAL ANALYSIS |
| Prerequisite        | Nil                |
| Credits (L-T-P)     | 3 (3-0-0)          |

- 1. introduce various numerical algorithm to find numerical solution of mathematical equation.
- 2. validate numerical solution through mathematical analysis.

#### **Course Content**

Linear Systems of Equations- Direct Methods-Gauss Jordan Elimination Method – Triangularization method – Cholesky method – Error Analysis - Iteration Methods - Jacobi iteration method – Gauss - Seidal iteration method – Eigen Value Problems: power method

Transcendental and polynomial equations-linear interpolation method-Mullar's method, Newton's method for non-linear system. Root squaring method, Bairstow method.

Interpolation and Approximation: Interpolating polynomials, divided differences-Spline curves. Least-Square approximation.

Numerical differentiation and integration-open and closed type formulae.

Numerical solution of ordinary differential equation- single and multi step methods for IVP. Stability analysis.

#### **Reference Books:**

- 1. David Kinciad & Ward Cheney, Numerical Analysis and mathematics of scientific computing, Brooks/Cole, 1999
- 2. K. Atkinson, Elementary Numerical Analysis, Jhon Wiley & Sons, 2004
- 3. Curtis E Gerald &Partrock O Whealtley, Applied Numerical Analysis, Pearson edu. 2004
- 4. M.K.Jain, S.R.K.Iyengar, R.K.Jain, NuericalMethodd:For Scientific and Engineering Computation, New Age International, 7<sup>th</sup>edn, 2019
- 5. John H. Mathews, Kurtis K. Fink, Nuerical Methods Using Matlab, 4<sup>th</sup>Edn, Pearson, 2004

- 1. find the numerical solution of linear system of equations AX = b
- 2. find the roots of transcendental and polynomial equations
- 3. approximate the function and interpolate function and its derivatives
- 4. find numerical differentiation of the function
- 5. find single and double integral numerically.
- 6. find numerical solution of ordinary differential equation.

| Course Code         | MA 7 12                          |
|---------------------|----------------------------------|
| Title of the Course | Statistical Computing-Laboratory |
| Prerequisite        | Nil                              |
| Credits (L-T-P)     | 2 (0-0-2)                        |

- 1. make the student learn statistical software and perform a number of statistical tests using R.
- 2. use built-in function to construct correlation and regression of given data.
- 3. perform statistical analysis over the large data.

## **Course Content**

Overview of R, R data types and objects, reading and writing data, Control structures, functions, scoping rules, dates and times, Loop functions, debugging tools, Simulation, code profiling.

#### **Reference Books:**

- 1. Kun Ren, Learning R Programming, Packt Publishing Ltd, 2016.
- 2. Colin Gillespie and Robin Lovelace, Efficient R Programming: A Practical Guide to Smarter Programming, "O'Reilly Media, Inc.", 2017

- 1. find statistical parameters(mean, median,...) for given lager data
- 2. find correlation coefficient between two variables,
- 3. find regression line & regression curve for large data
- 4. present graphical representation and analyze the data

| <b>Course Code</b>  | MA 7 13                 |
|---------------------|-------------------------|
| Title of the Course | Functional Analysis     |
| Prerequisite        | Real Analysis, Topology |
| Credits (L-T-P)     | 3 (3-0-0)               |

- 1. exposes the students to normed spaces, Banach spaces and Hilbert space by discussing the concepts of compactness, bounded linear operators.
- 2. introduces the various notions used in inner product space.
- 3. Introduces Hahn-Banach theorems, open mapping theorem, closed graph theorem, Riesz representation theorem are proved.

#### **Course Content**

Normed linear spaces. Metric induced by the norm. Cauchy's sequence, convergence of a sequence. Continuity of vector space operations and norm. Types of convergence of a series. Banach spaces. Finite dimensional normed linear spaces and subspaces. Equivalent norms and their properties. Compactness and finite dimension. Quotient space and products of normed linear spaces.

Bounded linear transformations. Space of Bounded linear transformations B[X, Y] and its properties. Continuous linear functionals and Duals of classical spaces. The Hahn-Banach extension and separation theorems. The open mapping theorem and its applications (The inverse mapping and closed graph theorems). The principle of uniform boundedness (Banach-Steinhaus theorem).

Inner product spaces. Norm induced by the inner product. Schwartz inequality, Parallelogram identity. Hilbert spaces. Relation between Banach and Hilbert spaces. Closest point in a closed convex subset. Projection theorem. Orthogonal complement of a subspace, Orthogonal decomposition. Orthogonal and orthonormal systems. Bessel's inequality, Parseval's identity. Characterizations of complete orthonormal systems. The Riesz representation theorem.

#### **Reference Books:**

- 1. B. Bollabas, Linear Analysis, Cambridge University Press (Indian edition),1999
- 2. J. B. Conway, A Course in functional Analysis, 2nd ed., Springer, 1985
- 3. E. Kreyszig, Introduction to Functional Analysis with Applications, Wiley, 1989
- 4. G. F. Simmons, Introduction to Topology and Modern Analysis, Krieger Publishing Co., 1983
- 5. A. E. Taylor and D. C. Lay, Introduction to Functional Analysis, 2nd ed., Wiley, New York, 1980

- 1. understand various concepts of normed spaces and construct new spaces from old ones.
- 2. find duals of standard spaces and also prove fundamental results about bounded linear functional.
- 3. find orthogonal decomposition of Hilbert spaces and prove several other results.
- 4. understand basics of various operators and their properties.

| Course Code         | MA 7 15               |
|---------------------|-----------------------|
| Title of the Course | Transforms Techniques |
| Prerequisite        | Nil                   |
| Credits (L-T-P)     | 3 (3-0-0)             |

- 1. introduce various transform technique to solve mathematical equation representing engineering problem.
- 2. discuss the properties of various integral transforms.
- 3. express periodic and non-periodic function in terms of sinusoidal functions.

#### **Course Content**

**LAPLACE TRANSFORMS-**Transforms of elementary functions - Properties - Differentiation and integration of transforms - Periodic functions - Initial & final value theorems - Inverse Laplace transforms - Convolution theorem - Error function - Transforms involving Bessel functions.

**FOURIER SERIES:** Dirichlet's Conditions – General Fourier Series – Half Range Sine Series – Half Range Cosine Series – Complex Form Of Fourier Series – Parseval's Identity – Harmonic Analysis.

**FOURIER TRANSFORMS-**Fourier integral representation - Fourier transform pairs - Properties - Fourier sine and cosine transforms - Transforms and inverse transforms of elementary functions - Convolution theorem - Transforms of derivatives.

**APPLICATIONS OF TRANSFORMS -** Application of Laplace Transforms - Evaluation of integrals - Solution of Linear ODE - Applications of Fourier Transforms - Heat equation on infinite and semi-infinite line - Potential problems in half-plane.

**Z- TRANSFORMS AND DIFFERENCE EQUATIONS-Z-**Transforms — Elementary Properties — Inverse Z — Transform (Using Partial Fraction And Residues) — Convolution Theorem — Formation Of Difference Equations — Solution Of Difference Equations Using Z — Transform.

## **Reference Books:**

- 1. Andrews, L.C. and Shivamoggi, B.K., "Integral Transforms for Engineers", SPIE press, 1999.
- 2. Sneddon, I.N., Fourier Transforms, Courier Corporation, 1995.
- 3. Andrews, L.C. and Shivamoggi, B.K., "Integral Transforms for Engineers and Applied Mathematicians", Mac Millan Publishing Co., 1988.
- 4. LokenathDebnath, Dambaru Bhatta, Integral Transforms and Their Applications, 3<sup>rd</sup>edn, CRC Press, 2016
- 5. Anthony C. Grove, An Introduction to the Laplace transform and the Z-transform, Prentice Hall, 1991.

- 1. find Laplace transform of functions and discuss various properties of Laplace transforms
- 2. express periodic functions in terms of sinusoidal functions
- 3. find Fourier transform of functions and discuss various properties of Fourier transforms
- 4. find Z-Transform transform of discrete functions and discuss various properties of Z-transforms
- 5. Apply transform to solve differential and difference equations.

| Course Code         | MA 7 17             |
|---------------------|---------------------|
| Title of the Course | Operations Research |
| Prerequisite        | Nil                 |
| Credits (L-T-P)     | 3 (3-0-0)           |

- 1. formulate various real-life problems as Operations Research models and to study methodologies to solve the problems.
- 2. Introduce Linear Programming, Transportation and Assignment problems and to discuss methods to find optimum solutions.
- 3. study the theory of duality and sensitivity analysis in linear programming.
- 4. learn network flow problems and their solution techniques.
- 5. explore dynamic programming problem and its applications.

#### **Course Content**

Introduction – Models in Operations Research – Linear Programming Problems – Simplex Method – Big-M Method – Two-Phase Method – Special Cases – Degeneracy and Cycling – Unbounded Solutions – Alternative Optima.

Dual Linear Programs – Duality Theorems – Dual Simplex Method - Transportation Problems – Finding an Initial Basic Feasible Solution - Optimality Condition – MODI method – Degeneracy – Assignment Problems – Hungarian Method.

Revised Simplex Method – Sensitivity Analysis – Parametric Programming.

Network Analysis – Shortest Route Problems – Maximal Flow Problems – Critical Path Method (CPM) – Program Evaluation and Review Techniques (PERT).

Dynamic Programming – Introduction – Principle of Optimality – Forward and Backward recursions – Discrete Dynamic Programming – Continuous Dynamic Programming – Applications.

#### **Reference Books:**

- 1. A. Ravindran, Don T. Phillips and James J. Solberg, *Operations Research- Principles and Practice*, John Wiley, 2014.
- 2. Hamdy A. Taha, *Operations Research-An Introduction*, Prentice Hall of India, 2000.
- 3. Frederick S. Hillier and Gerald J. Lieberman, *Introduction to Operations Research*, McGraw Hill, 2010.
- 4. KantiSwarup, P.K. Gupta and Man Mohan, *Operations Research*, Sultan Chand, 2014.

- solve Linear Programming Problem (LPP) using Simplex, Big-M and Two phase methods.
- find an optimum solution for transportation and assignment problems and to analyze LPP using duality results.
- solve LPP using Revised Simplex method and to apply duality methods in the study of sensitivity analysis in LPP and parametric programming.
- determine the shortest path, critical path and maximal flow in a network.

| Course Code         | MA 7 19                            |
|---------------------|------------------------------------|
| Title of the Course | Mathematical Software – Laboratory |
| Prerequisite        | Nil                                |
| Credits (L-T-P)     | 2 (0-0-2)                          |

- 1. introduce software available to solve mathematical problem.
- 2. understand and practice basic operators, functions available in SCILAB,
- 3. Perform symbolic calculations using SCILAB.
- 4. get hands-on-training in LATEX and learn drawing using LATEX.

#### **Course Content**

Introduction to SCILAB, Scalars & Vectors, Matrix operations, polynomials, plotting Functions and loops. String Handling Functions. Basic programming in Scilab

Elements of LaTeX; Hands-on-training of LaTex; graphics in LaTeX; PSTricks; Beamer presentation;

#### **Reference Books:**

- 1. L. Lamport. LATEX: A Document Preparation System, User's Guide and ReferenceManual. Addison-Wesley, New York, second edition, 1994.
- 2. David F. Griffiths, Desmond J. Higham, Learning LaTeX: Second Edition, SIAM, 2016.
- 3. Sandeep Nagar, Introduction to Scilab: For Engineers and Scientist, Apress, 2017
- 4. Tejas Sheth, Scilab: A Practical Introduction to Programming and Problem Solving, CreateSpace Independent Publishing Platform, 2016

- 1. perform basic operations and symbolic calculations in SCILAB.
- 2. Draw two and three dimensional graphs using SCILAB.
- 3. use in-built functions to solve problem in ODE, Linear algebra, trigonometry.
- 4. make documents, presentation and draw geometries using LATEX.

| Course Code         | MA 7 21      |
|---------------------|--------------|
| Title of the Course | GRAPH THEORY |
| Prerequisite        | Nil          |
| Credits (L-T-P)     | 3 (3-0-0)    |

## **Course Learning Objectives:**

• To have general awareness of some applications oriented concepts in Graph Theory and apply them as a tool in the various problems in different networks models.

#### **Course Content**

Introduction various types of graphs and applications- Definitions, examples and some results related to degree of vertex, walk, trail, path, tour, cycle traversals in a graph, complement of a graph, self-complementary graph, connectedness and connectivity graph, distance, shortest path in a graph, radius and diameter of a graph and various special graphs such as complete, cycles, bipartite, k-partite(with respect to some property). Some basic eccentric properties of graphs, tree, spanning tree, coding of spanning tree. Number of spanning trees in a complete graph. Recursive procedure to find number of spanning trees. Construction of spanning trees.

Directed graphs: Definitions and examples of strongly, weakly, unilaterally connected digraphs, strong components and its applications. Matrix representation of graph and digraphs and some properties (proof not expected). Properties of Eulerian graphs and its applications, Hamiltonian graph-standard theorems and applications (Dirac theorem, Chavtal theorem, closure of graph). Non Hamiltonian graph with maximum number of edges. Self-centered graphs, dominating edge in a graph and some results related to eccentricity properties of complement of a graph and its applications.

Chromatic number; vertex chromatic number of a graph, edge chromatic number of a graph (only properties and examples)-applications to colouring. Planar graphs, Euler's formula, maximum number of edges in a planar graph, dual of planar embedding, some problems related to planarity and non-planarity, Five colour theorem(proof). Four colour problem and its related results, Vertex Covering, Edge Covering, Vertex independence number, Edge independence number, relation between them and number of vertices of a graph.

Matching theory, maximal matching, algorithms for maximal matching and some results related to matching in bi-partite graphs and its applications. Perfect matching (only properties in general graph and applications to regular graphs). Some properties of Tournaments, some results on strongly connected tournaments. Notion of domination in graphs and various domination parameters in graph and their applications real life problems.

BFS and DFS algorithms and their applications, shortest path algorithm, Min-spanning tree and Max-spanning tree algorithms and their applications, Planarity algorithm. Flows in graphs; Maxflow-Mincut theorem, algorithm for maxflow. CPM in directed networks. Time complexity of all above polynomial time algorithms and other type problems; P-NP-NPC-NP hard problems and examples.

#### **Reference Books:**

- 1. J.A.Bondy and U.S.R.Murthy, Graph Theory with Applications, Macmillan, London (1976) EBook, Freely Downloadable.
- 2. Cormen, Leiserson, Rivest, and Stein, Introduction to Algorithms (Second edition), McGraw-Hill (2001).
- 3. M.Gondran and M.Minoux: Graphs and Algorithms, John Wiley, 1984.
- 4. Fred Buckley and Frank Harary, Distance in Graphs, Addison Wesley Publishing Company, Newyork, 1989.
- 5. T.W.Heynes, S.T. Hedetniemi, P.J.Slater, Fundementals of domination in graphs, Marcel Dekker, Inc. Newyork, 1998a.
- 6. T.W.Heynes, S.T. Hedetniemi, P.J.Slater, Domination in graphs, Advanced topics of domination in graphs, Marcel Dekker, Inc. Newyork, 1998b.

## **Course Learning Outcomes:**

- 1. understand the various types of graphs, graph properties and give examples for the given property.
- 2. model the given problem from their field to underlying graph model.
- 3. proceed to solve the problem either through approximation algorithm or exact algorithm depending on the problem nature.
- 4. appreciate the applications of digraphs and graphs in various communication networks
- 5. appreciate the applications of graphs and digraphs in various other fields.

| Course Code         | MA 7 23        |
|---------------------|----------------|
| Title of the Course | Measure Theory |
| Prerequisite        | Real Analysis  |
| Credits (L-T-P)     | 3 (3-0-0)      |

#### **Course Learning Objectives:**

This course exposes the students to Lebesgue measure and Lebesgue integral. It also introduces the convergence theorems involving integration, absolute continuity and functions of bounded variation.

#### **Course Content**

Ring and algebra of sets, sigma-algebras. Borel algebra and Borel sets. Lebesgue outer measure on R, countable sub-additivity, measurable set, sigma-algebra structure of measurable sets-countable additivity of Lebesgue measure on R, Cantor set. Construction of a non-measurable subset of [0, 1], measurable functions, Approximation of measurable functions, Egorov's theorem, Lusin's theorem.

Lebesgue integral of non-negative measurable functions, Integrable functions and Lebesgue integral of integrable functions, linearity, Monotone convergence theorem, Fatou's lemma, Dominated convergence theorem. Comparison of Riemann and Lebesgue integration, Lebesgue integrability of Riemann integrable functions, characterization of Riemann integrable functions, improper Riemann integrals and their Lebesgue integrals.

Functions of bounded variation, Indefinite integrals of Lebesgue integrable functions on [a, b], statement of Vitali's lemma and almost everywhere differentiability of monotone increasing functions. Absolutely continuous functions, properties, absolute continuity of indefinite integral of Lebesgue integrable functions, Differentiation of indefinite integrals, characterization of absolutely continuous functions as indefinite integrals.

Completeness of  $L_p(\mu)$  spaces. Signed measures, Hahn and Jordan decompositions, Radan-Nikodym theorem (without proof).

#### **Reference Books:**

- 1. G. de Barra, Measure and Integration, Wiley Eastern, 1981.
- 2. H. L. Royden, P. M. Fitzpatrick, Real Analysis, 4th ed., Pearson education, 2011
- 3. W. Rudin, Real and Complex Analysis, 3rd ed., McGraw-Hill International Editions, 1987.

- 1. understand the concept of Lebesgue measure, measurable sets and approximation of a measurable function by simple measurable function.
- 2. prove various convergence theorems and know applications of them.
- 3. understand the absolutely continuous functions and functions of bounded variation.
- 4. do decomposition of measures.

| Course Code         | MA 7 25                             |
|---------------------|-------------------------------------|
| Title of the Course | Sobolov spaces and its Applications |
| Prerequisite        | Functional Analysis                 |
| Credits (L-T-P)     | 3 (3-0-0)                           |

- 1. The knowledge of functional analysis in real life problems
- 2. Existence and Uniqueness of regular solutions.

#### **Course Content**

*Introduction:* Elements of operator theory and Hilbert spaces.

*Distribution Theory:* Test functions and distributions, Convolution of Distributions, Tempered Distributions.

*Sobolev Spaces:* Definition and basic properties, Extension Theorems, Imbedding theorems, Compactness theorems, Trace theory.

Weak Solutions of Elliptic boundary value problems: Examples of Elliptic BVPs, Existence and Regularity of weak solutions, Maximum principle, Eigenvalue problems.

#### **Reference Books:**

- 1. S. Kesavan, *Topics in Functional Analysis and Applications*, New Age International Publishers, 2015.
- 2. H. Brezis, Functional Analysis, Sobolev Spaces and Partial Differential Equations, Springer, 2011.
- 3. M. Renardy and R. C. Rogers, *An Introduction to Partial Differential Equations*, Springer, 2004.

#### **Course Learning Outcomes:**

completion of the course, students should be able to

- 1. identify Sovolov spaces
- 2. apply functional analysis to study real life problems (existence, uniqueness of regular solutions).

| Course Code         | MA 7 27                                      |
|---------------------|--|
| Title of the Course | Numerical Solution of Differential Equations |
| Prerequisite        | Nil  |
| Credits (L-T-P)     | 3 (3-0-0)                                    |

- 1. give an understanding of numerical methods for the solution boundary value problem of ordinary differential equations, their derivation, analysis and applicability
- 2. Introduce finite difference methods for partial differential equations (PDEs)
- 3. discuss stability, consistency and convergence of the scheme for initial and initial boundary value problems.
- 4. basic idea of finite element analysis.

#### **Course Content**

**Ordinary differential equations-**Boundary-value problems-shooting method, finite difference methods – convergence analysis.

**Parabolic equation:** One dimensional parabolic equations -Explicit and implicit finite difference scheme Stability and convergence of difference scheme. Two dimensional parabolic equations - A.D.I. methods with error analysis.

**Hyperbolic equations-**First order quasi-linear equations and characteristics - Numerical integration along a characteristic - Lax-Wendroff explicit method - Second order quasi-linear hyperbolic equation - Characteristics - Solution by the method of characteristics.

**Elliptic equations-**Solution of Laplace and Poisson equations in a rectangular region - Finite difference in Polar coordinate Formulas for derivatives near a curved boundary when using a square mesh - Discretisation error - Mixed Boundary value problems.

**Finite Element Method**: types of integral formulations, one and two dimensional elements, Galerkin formulation, application to Dirichlet and Neumann problems.

#### **Reference Books:**

- 1. G. Evans, J. Blackledge, P. Yardley, Numerical Methods for Partial Differential Equations, Springer Science & Business Media, 2012
- 2. K. W. Morton, D. F. Mayers, Numerical Solution of Partial Differential Equations: An Introduction, Cambridge University Press, 2005
- 3. John A. Trangenstein, Numerical Solution of Elliptic and Parabolic Partial Differential Equations, Cambridge University Press, 2013.
- 4. M.K.Jain, Numerical solution of Differential Equations, New Age International Publishers, 2008
- 5. J.N. Reddy, An Introduction to Nonlinear Finite Element Analysis: With Applications to Heat Transfer, Fluid Mechanics, and Solid Mechanics, Oxford University Press, 2015.

- 1. Solve the boundary value problem in ordinary differential equations
- 2. Write finite difference numerical scheme for parabolic, elliptic and hyperbolic equations and solve numerically.
- 3. discuss the convergence of numerical scheme for differential equations.
- 4. apply finite element method for differential equations.

| Course Code         | MA 7 29        |
|---------------------|----------------|
| Title of the Course | Fluid Dynamics |
| Prerequisite        | Nil            |
| Credits (L-T-P)     | 3 (3-0-0)      |

- 1. understand physics involve in fluid flow problems and apply laws of conservation to construct mathematical model.
- 2. find mathematical solution of some viscous and inviscid flow problems

#### **Course Content**

Real Fluids and Ideal Fluids - Streamlines and Path lines; Steady and Unsteady Flows - The Velocity potential – The Vorticity vector - The Equation of continuity - Acceleration of a Fluid – Conditions at a rigid boundary - General analysis of fluid motion - - Euler's equations of motion - Bernoulli's Equation.

Discussion of a case of steady motion under conservative body forces – Some potential theorems- Some Flows Involving Axial Symmetry - Some special two- Dimensional Flows - Impulsive Motion. Some three-dimensional Flows: Introduction - Sources, Sinks and Doublets - Images in a Rigid Infinite Plane - Axi-Symmetric Flows; Stokes stream function

Two-Dimensional Flows: The stream function – The Complex Potential for Two-Dimensional, Irrotational, Incompressible Flow - complex velocity potentials for Standard Two-Dimensional Flows - The Milne-Thomson circle theorem and applications – The Theorem of Blasius

Viscous flow: Stress components in a Real fluid - relations between Cartesian components of stress - Translational Motion of Fluid Element - The Rate of Strain Quadric and Principal Stresses - Some Further properties of the Rate of Strain Quadric - Stress Analysis in Fluid Motion - Relations Between stress and rate of strain - The Navier - Stokes equations of Motion of a Viscous Fluid .

Some exact solutions of Viscous Flow - Steady Viscous Flow in Tubes of Uniform cross section - Diffusion of Vorticity - Energy Dissipation due to Viscosity - Steady Flow past a Fixed Sphere - Dimensional Analysis; Reynolds Number - Prandtl's Boundary Layer.

#### **Reference Books:**

- 1. F. Chorlton, Text book of fluid dynamics, CBS Publishers & Distributors, 2005
- 2. J.D. Anderson, Computational Fluid Dynamics, The Basics with Applications, McGraw Hill, 2012.
- 3. G.K. Batchelor, An Introduction to Fluid dynamics, Cambridge University Press, 2000.
- 4. Richard E. Meyer, Introduction to Mathematical Fluid Dynamics, Courier corporation, 2012
- 5. A.J. Chorin and A. Marsden, A Mathematical Introduction to Fluid Dynamics, Springer Science & Business media, 2013.

- 1. understand physical concept involved in fluid motion
- 2. model some two and three dimensional flows of viscous and inviscid fluid flows.
- 3. find mathematical solution of some fluid flow problems and interpret results physically.

| Course Code         | MA 7 16              |
|---------------------|----------------------|
| Title of the Course | Hardy-Hilbert Spaces |
| Prerequisite        | Functional Analysis  |
| Credits (L-T-P)     | 3 (3-0-0)            |

## **Course Learning Objectives:**

This course introduces various concepts related to Hardy-Hilbert Space. It includes a study of shift operators, invariant subspaces and inner and outer functions. It also deals with Toeplitz, Hankel and Composition operators.

#### **Course Content**

The Hardy Hilbert Space: Basic Definitions and properties. The unilateral shift and factorisation of Spectral structure. functions: Shift operators, Invariant and reducing subspaces.

Inner and outer factorisation, Blaschke factors, singular inner functions, outer functions. Toeplitz operators: Basic properties of Toeplitz operators, spectral structure.

Hankel operators: Bounded Hankel operators, Hankel operators of finite rank, Compact Hankel operators, self adjointness and normality of Hankel operators. Relation between Hankel and Toeplitz operators.

Composition Operators: Fundamental Properties of Composition Operators, Invertibility of Composition Operators, Eigenvalues and Eigenvectors, Composition Operators Induced by Disk Automorphisms.

#### **Reference Books:**

- 1. R.A. Martinez-Avedano and P. Rosenthal, An Introduction to the Hardy-Hilbert Space, Graduate Texts in Mathematics 237, Springer, 2007.
- 2. R.G. Douglas, Banach Algebra Techniques in Operator Theory, Graduate Texts in Mathematics 179, Springer, 1998
- 3. N.K. Nikolskii, Operators, Functions and Systems: An Easy Reading, Volume 1, Mathematical Surveys and Monographs 92, American Mathematical Society, 2002

- 1. identity Hardy-Hilbert spaces in several equivalent forms and know various properties of shift operators.
- 2. prove theorems about Hankel operators and Toeplitz operators
- 3. prove theorems about Composition operators.

| Course Code         | MA 7 18                            |
|---------------------|------------------------------------|
| Title of the Course | Operator Theory                    |
| Prerequisite        | Real Analysis, Functional Analysis |
| Credits (L-T-P)     | 3 (3-0-0)                          |

#### **Course Learning Objectives**: This course

- 1. introduces the dual representation of classical Banach spaces and necessary notions of convergence in Banach spaces.
- 2. also gives a detailed exposure of various operators on Banach spaces and Hilbert spaces.

#### **Course Content**

Dual space considerations: Representation of duals of the spaces  $c_0$ 0 with p-norms,  $c_0$  and c with supremum-norm,  $l_p$ , C[a,b] and  $L_p$ . Reflexivity, weak and weak\* convergences.

Adjoint, self adjoint, normal and unitary operators on Hilbert spaces and their properties. Projection operators on Banach spaces and Hilbert spaces. Eigenvalues, Eigenvectors and Eigen spaces, Invariant spaces, Spectral theorem on finite dimensional Hilbert spaces.

Operators on Banach and Hilbert spaces: Compact operators and its properties; Integral operators as compact operators; Adjoint of operators between Hilbert spaces; Self-adjoint, normal and unitary operators; Numerical range and numerical radius; Hilbert-Schmidt operators.

Spectral results for Banach and Hilbert space operators: Eigen spectrum, Approximate eigen spectrum, Spectrum and resolvent; Spectral radius formula; Spectral mapping theorem; Riesz-Schauder theory, Spectral results for normal, Self-adjoint and unitary operators; Functions of self-adjoint operators.

#### **Reference Books:**

- 1. J. B. Conway, A Course in Functional Analysis, 2nd ed., Springer, 1985
- 2. J. B. Conway, A Course in Operator Theory, Graduate studies in Mathematics, vol. 21, AMS, 2000
- 3. R. G. Douglas, Banach Algebra Techniques in Operator Theory, Graduate Texts in Mathematics, vol.179, Springer-Verlag, 1998.
- 4. E. Kreyszig, Introduction to Functional Analysis with Applications, Wiley, 1989.

- analyse various convergence notions in Banach spaces.
- find the eigen spectrum, resolvent and spectral radius of operators.
- prove the spectral results of specific kind of operators.
- find representation of compact self-adjoint operators.

| Course Code         | MA 7 20                                |
|---------------------|--|
| Title of the Course | FUZZY MATHEMATICS AND ITS APPLICATIONS |
| Prerequisite        | nil                                    |
| Credits (L-T-P)     | 3 (3-0-0)                              |

## **Course Learning Objectives:**

To enable the students to understand the concept of fuzzy logic, fuzzy sets, properties of  $\alpha$ -cuts, extension principles, fuzzy complements, fuzzy intersection, fuzzy union, fuzzy numbers, fuzzy relations, fuzzy equivalence relations, classifications by fuzzy equivalence relations, c-means clustering, fuzzy c-mean clustering, fuzzy decision making methods and their applications.

#### **Course Content**

Fuzzy sets – introduction, Basic types and Basic concepts, Additional properties of  $\alpha$ -cuts, Representation of fuzzy sets, Extension principles.

Type of operators on fuzzy sets and fuzzy complements, Fuzzy intersection and fuzzy unions, Combination of operations.

Fuzzy numbers and arithmetic operations on intervals, Arithmetic operations on fuzzy numbers, Fuzzy equations and fuzzy relations, Binary fuzzy relations and binary relation on a single set, Fuzzy equivalence relations.

Classification by equivalence relations-Crisp relations, Fuzzy relations, Cluster Analysis, Cluster Validity, c-means Clustering- Hard c-means(HCM), Fuzzy c-Means(FCM).

Fuzzy Decision making – introduction, Conversion of linguistic variables to fuzzy numbers, Individual Decision Making, Multiperson decision Making, Multicriteria decision Making, Fuzzy ranking nethods.

#### **Reference Books:**

- 1. George J.Klir, Bo Yuan, Fuzzy Sets and Fuzzy logic Theory and Applications, Prentice Hall India, New Delhi, 1997.
- 2. H.J Zimmermann, Fuzzy sets, Decision making and expert systems, Kluwer, Bosten, 1987.
- 3. S.J. Chen and C.L.Hwang, Fuzzy Multiple Attributes Decision Making, Springer verlag, Berlin Heidelberg, 1992.

#### **Course Learning Outcomes:**

Upon successful completion of this course, students will be able to

- 1. analyze the types of fuzzy sets,  $\alpha$ -cuts and its properties and extension of functions.
- 2. apply the operations (fuzzy complements, fuzzy intersections and fuzzy unions) on fuzzy sets.
- 3. create the fuzzy relations and identify the different types of fuzzy relations and their applications.
- 4. apply the concepts of clustering of data (information) in engineering problems.
- 5. apply the concepts of fuzzy decision making methods in engineering and management problems.

| Course Code         | MA 7 22                              |
|---------------------|--------------------------------------|
| Title of the Course | Asymptotics and Perturbation Methods |
| Prerequisite        | Ordinary Differential Equations      |
| Credits (L-T-P)     | 3 (3-0-0)                            |

## **Course Learning Objectives:**

The overarching goal of this course is to introduce the fundamentals of asymptotic analysis and perturbation methods for solving problems arising in the study of differential equations and integrals

#### **Course Content**

Introduction to asymptotic approximations: Definitions; Convergence; Asymptoticness; Parametric expansions. Asymptotic analysis of integrals: Solutions to algebraic equations: Regular and singular perturbations;

Regular perturbation problems in ODEs and PDEs: Initial value problems; Boundary perturbations. Introduction to singular perturbation of ODEs. Poincare-Lindstedt method.

Boundary layer theory, WKB Theory, Multiple-scale analysis, Introduction to singular perturbation of PDEs, Engineering applications: At least one example each from fluid mechanics, solid mechanics, and vibrations.

#### **Reference Books:**

- 1. Hinch, E. J., 1991. Perturbation Methods, Cambridge U. Press: Cambridge, U.K.
- 2. Murdock, J. A., 1987. Perturbations: Theory and Methods, SIAM.
- 3. Van Dyke, M., 1975. Perturbation Methods in Fluid Mechanics. Parabolic Press.
- 4. Kevorkian, J., and J. D. Cole, 1981. Perturbation Methods in Applied Mathematics. Springer
- 5. Holmes, M. H. 2013. Introduction to Perturbation Methods. Springer.
- 6. D. Wilcox (1995) Perturbation methods in the computer age. DWC Industries Inc.
- 7. Bender & S. Orszag (2010) Advanced mathematical methods for scientists and engineers. Springer

- 1. Explain basic concepts of perturbation techniques, such as order relationships, asymptotic sequences, asymptotic expansions and convergence issues.
- 2. Propose a solution method for regular perturbation problems
- 3. Explain the difference between a regular and a singular perturbation problem
- 4. Analyze a singular problem by means of a balancing method
- 5. Determine inner and outer solutions for singular perturbation problems by means of boundary-layer theory and the composite form
- 6. Use WKB methods to solve linear ordinary differential equations subjected to different length or time scales
- **7.** Perform a multiple-scale analysis on linear and non-linear problems Apply perturbation methods to partial-differential problems

| Course Code         | MA 7 24                                       |
|---------------------|---|
| Title of the Course | Integral Equations and Calculus of Variations |
| Prerequisite        | Nil   |
| Credits (L-T-P)     | 3 (3-0-0)                                     |

- 1. different types of Integral equations arising in several engineering sectors, and methods/tools for solving different types of integral equations
- 2. the concept of resolvent kernels, Eigenvalues and eigenvectors of Fredholm integral equtions, Green's function for a boundary value problem
- 3. necessity of calculus of variations in physical aspects (different problems and their method of solutions).
- 4. Galerkin and Collocation methods to obtain approximate solutions.

#### **Course Content**

**Integral Equations**: Basic concepts, Volterra integral equations, Relationship between linear differential equations and Volterra equations, Resolvent kernel, Method of successive approximations, Convolution type equations, Volterra equation of the first kind. Abel's integral equation. Fredholm integral equations, Fredholm equations of the second kind, The method of Fredholm determinants, Iterated kernels, Integral equations with degenerate kernels, Eigenvalues and Eigen functions of a Fredholm alternative, Construction of Green's function for BVP, Singular integral equations.

**Calculus of Variations**: Euler – Lagrange equations, Degenerate Euler equations, Natural boundary conditions, Transversality conditions, Simple applications of variational principle, Sufficient conditions for extremum. Variational formulation of BVP, Minimum of quadratic functional. Approximate methods – Galerkin's method, Weighted-residual methods, Collocation methods. Variational methods for time dependent problems.

#### Reference Books:

- 1. David Porter, and David S.G. Stirling, *Integral Equations: A Practical Treatment, from Spectral Theory to Applications*, Cambridge texts in Applied Mathematics, 1990.
- 2. Ram P. Kanwal, *Linear Integral Equations: Theory and Technique*, Springer.
- 3. Abdul–Majid Wazwaz, *Linear and Non-linear Integral Equations: Methods and Applications*. Springer.
- 4. Robert Weinstock, *Calculus of Variations (with Applications to Physics and Engineering)*, Dover Publications, INC.
- 5. M. Gelfand, and S. V. Fomin, *Calculus of Variations*. Prentice–Hall, INC.

- 1. identify different integral equations and classify them.
- 2. find solution/approximate solutions, using resolvent kernels, iterated kernels etc.
- 3. construct Green's function for a boundary value problem.
- 4. solve Euler Lagrange problems and justify several aspects of the solution.
- 5. solve differential equation with several dependent variables.
- 6. apply Galerkin and Collocation methods to find approximate solutions of different problems.

| Course Code         | MA 7 26                |
|---------------------|------------------------|
| Title of the Course | Non-Linear Programming |
| Prerequisite        | Nil                    |
| Credits (L-T-P)     | 3 (3-0-0)              |

- 1. introduce the concepts of convex functions and to explore convex programming problems.
- 2. discuss the optimality conditions for the constrained and unconstrained nonlinear programming problems.
- 3. study the algorithms to solve nonlinear programming.
- 4. learn the theory of duality in nonlinear programming.

#### **Course Content**

Problem statement – Local and Global optimality – Convex sets – Convex functions and their properties – Differentiable convex functions – Convex Programming problems.

Unconstrained optimization of functions of several variables – First-order and second-order Optimality conditions – The method of Steepest Descent – Newton's method – The conjugate gradient method.

Constrained nonlinear optimization problems – Equality and inequality constraints – Method of Lagrange multipliers – Karush-Kuhn-Tucker conditions.

Algorithms for constrained optimization - Frank-Wolfe's method - Projected Gradient methods - Beale's method - Penalty methods.

Duality in nonlinear programming – Lagrangian Dual problem – Duality Theorems – Saddle point optimality conditions – Special cases – Linear and Quadratic Programming.

#### Reference Books:

- 1. Mokhtar S. Bazaraa, Hanif D. Sherali and C. M. Shetty, *Nonlinear Programming: Theory and Algorithms*, John Wiley, 2013.
- 2. Edwin K. P. Chong and Stanislaw H. Zak, *An Introduction to Optimization*, John Wiley, 2004.
- 3. David G. Luenberger and Yinyu Ye, *Linear and Nonlinear Programming*, Springer, 2008.
- 4. Singiresu S. Rao, Optimization: Theory and Application, John-Wiley, 2009.
- 5. Suresh Chandra, Jeyadeva and Aparna Mehra, *Numerical Optimization with Applications*, Narosa Publishing House, 2009.

- 1. determine convex sets and convex functions and to solve convex programming problems.
- 2. verify the optimality conditions and to apply Steepest Descent, Newton's and conjugate gradient methods to solve unconstrained optimization problems.
- 3. use the method of Lagrange multipliers and Karush-Kuhn-Tucker conditions to find an optimum solution of a nonlinear programming problems.
- 4. solve constrained nonlinear programming using Frank-Wolf's, Projected Gradient, Beale and Penalty methods.
- 5. construct the dual of a non-linear programming problem and to apply duality results to solve them.

#### PROJECT WORK

Project work: Project work has 8 credits. The duration of the project work is one semester. Candidates can do the project work on recent research problem or latest research articles published in reputed international journals. The work can be done in the department itself or candidates can go to reputed department/institution which has MOU with NITT.